National Aeronautics and Space Administration

Educational Product	
Educators & Students	Grades 3–12

EB-2001-12-017-JPL

Educational Brief

CASSINI SCIENCE INVESTIGATION

Planetary Billiards

Objective

To illustrate how the force of gravity is used to modify the trajectory of a spacecraft.

Time Required: 1-3 hours, depending on activities selected

Saturn System Analogy: Cassini's tour of the Saturn system using Titan for gravity-assist trajectory modifications.

Keywords: Angle, Asymptote, Eccentricity, Flyby, Gravity Assist, Hyperbola, Inclination, Vector

MATERIALS

1. Trajectory Modification Demonstration

- Strong magnet (available at hardware stores)
- Steel bearing balls (available at hardware stores, an auto parts store, bicycle shop, or surplus house). *Get several different sizes (1/8" and larger) of magnetic balls and spares (the balls are easy to lose!).*
- A piece of transparent Plexiglas (available at hardware stores) for a baseboard (approximately 10" square). *Size and shape are not important as long as there is room for the balls to roll; the Plexiglas should be flat and not bend easily. Any thin, nonmagnetic material will work as a base board, but targeting the balls is easier if the magnet is visible. The thickness of the material must be such that a magnet will have a noticeable affect on the balls rolling on top of the baseboard above it.*
- 6"–12" piece of angle aluminum—a right-angle V cross-section (available at hardware stores)

2. Effect on Trajectory Demonstration

- Laser pointer (available at camera stores and department stores). *A flashlight could be used, but the spread in the beam makes the effect more difficult to observe and measure and a larger mirror (see next item) is necessary.*
- Make-up mirror (or other small mirror)
- Alternative materials: rubber "super" balls (available at a toy store) and a solid wall in the classroom; plain white wrapping paper for recording data.

Discussion

Sending spacecraft to the planets is a complicated process. It requires careful study of such factors as:

- Payload — Science investigations that will be carried out; science instruments, plus their supporting hardware, electronics, and power supplies; and rocket engines and their propellants necessary to accomplish the goals of the program.
- Booster rocket lift capability — How much a rocket can carry into space beyond Earth's gravitational influence.
- Flight time — How long the mission will take, with factors such as hardware lifetime affecting the choices.

The Cassini–Huygens spacecraft embarked on October 15, 1997, on a mission to Saturn that will continue through

June 2008. The first 6-3/4 years are called the cruise phase (Figure 1).

The last four years involve a "tour" of the Saturn system (Figure 2), including detailed studies of the planet, rings, the large satellite Titan and the numerous smaller satellites, and Saturn's extensive magnetosphere (the volume of space that is filled with atoms and electrically charged particles and controlled by the magnetic field generated by the planet). Close flybys of Titan provide good opportunities to study that satellite. Flybys use the satellite's gravity to change the eccentricity (shape), inclination with respect to Saturn's equator, and solar orientation of the spacecraft's orbit so it can travel widely throughout Saturn's environment.

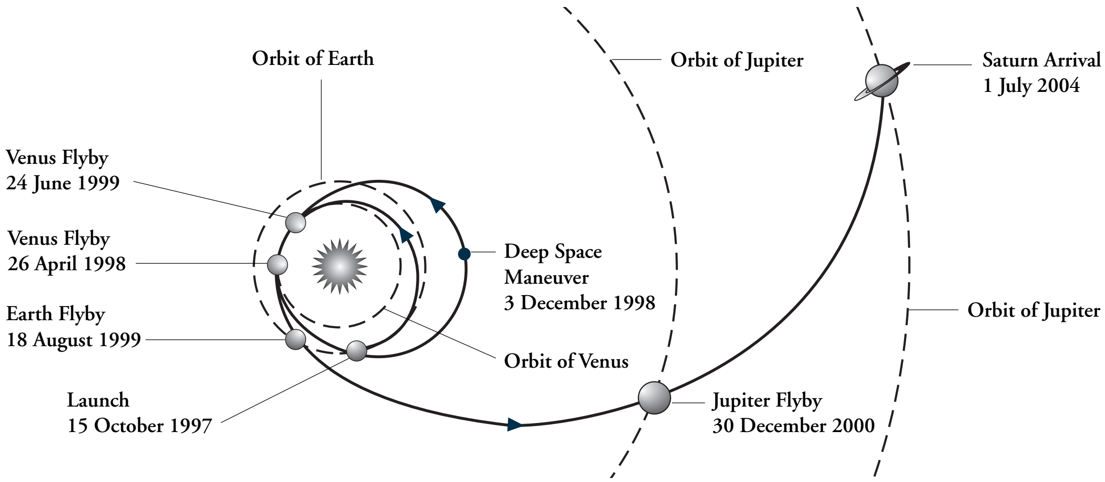
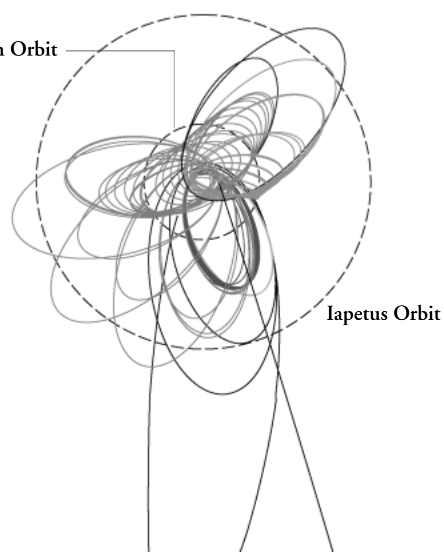


Figure 1. The cruise phase: Cassini–Huygens' route to Saturn is shown, with major milestones indicated by their dates. The Deep Space Maneuver uses the spacecraft's large rocket engine to adjust its path. The planetary flybys also adjust the path, but don't require the engine to be used. On the scale of this diagram, the change in direction due to the flybys cannot be seen.



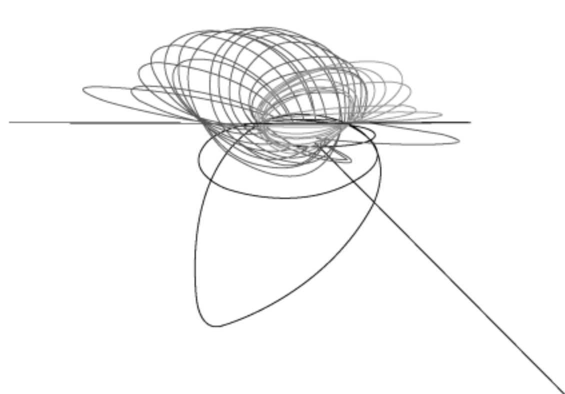


Figure 2. The Cassini orbiter's "tour" of Saturn's system is traced with orbital tracks around Saturn. The left diagram is a "petal plot" with the view looking down on Saturn's north pole. The right diagram shows orbit tracks as seen from the plane of Saturn's equator and rings. Note that the orbiter ranges widely around Saturn and above and below the planet's poles.

Because exploration requirements demanded a very capable spacecraft — one with a wide variety of advanced instrumentation and therefore very massive — the largest expendable rocket in the U.S. fleet, the Titan IVB Centaur SRMU, was chosen to launch the spacecraft. The Titan IV is the fourth generation in a family of launchers first designed in the 1950s. The B refers to an upgrade from the A model of this generation of Titans. SRMU stands for Solid Rocket Motor Upgrade — advanced, lighter weight, and more powerful solid-fuel rocket motors that strap onto the liquid-fueled Titan core. This portion of the launcher lifted the spacecraft into Earth orbit with the help of the Centaur rocket.

The Centaur is a liquid-fueled rocket that finished the Titan's job of boosting Cassini–Huygens into Earth orbit. It then ignited again after "coasting" 1/3 of the way around the world and propelled Cassini–Huygens out of Earth orbit and into a solar orbit through interplanetary space.

As large as the Titan IVB Centaur SRMU is, it could not send Cassini–Huygens directly to Saturn. Instead, the spacecraft was directed onto a trajectory involving a series of orbits around the Sun. These orbits include flybys of three planets: Venus (twice), Earth, and Jupiter. Using the gravity of the planets to change the speed and direction of the spacecraft, the combination of these flybys supplies the equivalent of the rocket power of the Titan launcher, while using a minimal amount of Cassini's onboard propellants.

The exercises described here will give students an understanding of the mechanics of planetary flybys through first-hand observations and measurement. In addition, students can use vectors to understand how planetary flybys can shorten interplanetary cruise durations.

The force of gravity from a body actually extends to infinity. But practically speaking for interplanetary flight, the gravitational force affecting a spacecraft is limited in extent. Around the time of a planetary flyby, a spacecraft can be characterized as following a curve called a hyperbola (Figure 3). When it is far from the planet, the spacecraft's path parallels the asymptote, a straight line that is very close to the actual hyperbolic path far from the focus (planet).

(Note: This is not true, strictly speaking. A spacecraft on the way to a planetary flyby is actually on an elliptical, heliocentric orbit. Over short distances [on an astronomical scale], a portion of the ellipse can look like a straight line, specifically outside the range where a planet affects the spacecraft's trajectory — less than a few hundred thousand kilometers in the case of a planet with approximately Earth's mass.)

As it gets closer to the planet, the path of the spacecraft deviates more and more from the asymptote, following the hyperbolic trajectory, and the spacecraft speeds up. After the flyby the spacecraft slows down to the same speed (relative to the planet) that it had inbound, and joins the outbound asymptote.

The amount of "bending" is a function of the spacecraft's flyby speed and mass, the planet's mass, and the distance of the close approach from the planet's center. Figure 3 shows two hyperbolas to indicate the variation in bending depending on the distance from the planet.

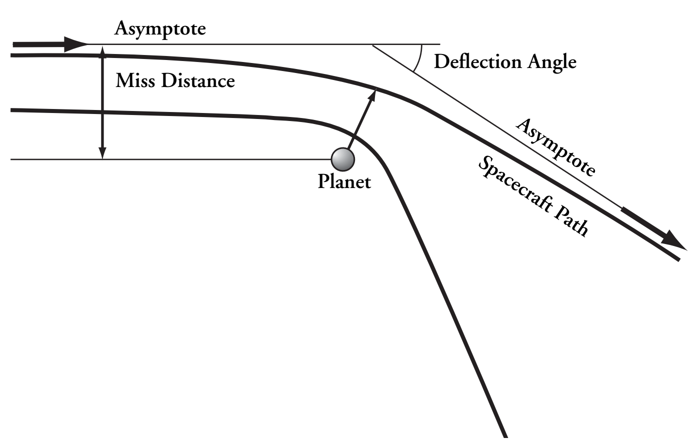


Figure 3. The deflection of a spacecraft going by a planet differs depending on its miss distance, its speed, and the mass of the planet. These same parameters apply when demonstrating the effect of a magnet on steel balls rolling by (the strength of the magnet is analogous to the mass of the planet). Two hyperbolas are shown here, for near and more distant flybys. The miss distance is illustrated for the more distant flyby and is the separation of the asymptote from the planet's center.

The speed of the spacecraft on the asymptote before the flyby is "v-infinity" (the speed when not influenced by the gravity of the flyby planet, typically 35 to 41 kilometers/second at Venus and Earth, 12 kilometers/second at Jupiter, and 6 kilometers/second at Saturn's moon Titan), and the speed is v-infinity afterwards (Figure 4). The inbound speed increase caused by the planet's gravity is exactly decreased during the outbound leg. However, the spacecraft's direction of motion is altered — the spacecraft's velocity (combined speed and direction) relative to the Sun changes. At the completion of a flyby the spacecraft's direction has changed, but not its speed. It is the change in velocity relative to the Sun that makes flybys useful in shortening interplanetary flights.

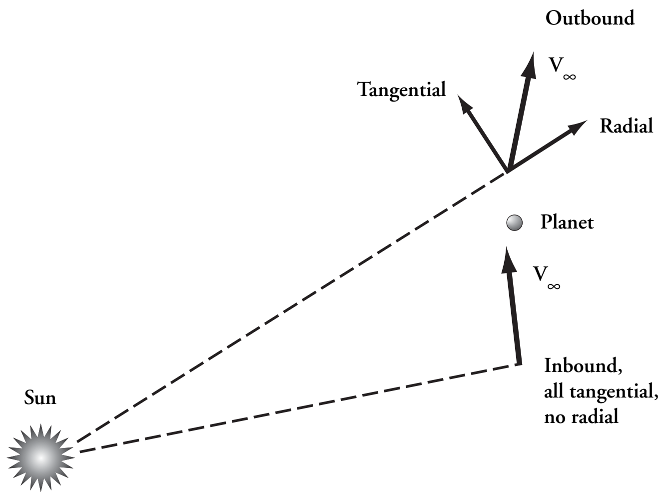


Figure 4. This diagram exaggerates the effect of a planetary flyby. In this example, inbound (before the planetary flyby), the spacecraft's velocity is perpendicular to a line from the Sun (called a radius vector). It is only moving around the Sun (with tangential velocity, but no radial velocity), not approaching or receding from it. After the flyby, the spacecraft's overall speed is the same (note that the bold vectors have the same length before and after) but the outbound vector can be resolved into tangential and radial components: the tangential component is smaller after the flyby and the radial component is large, when it had been zero before. In reality, the radial component typically changes only a few kilometers/second compared to a v-infinity of a few tens of kilometers/second.

Procedure

1. Trajectory Modification Demonstration

Rest the baseboard on the magnet, approximately centered. Use any convenient spacers to prop up the corners of the baseboard so it stays flat and level (Figure 5). The launch ramp can be mounted on the side of a small box so the angle remains consistent. Incline the launcher at a very shallow

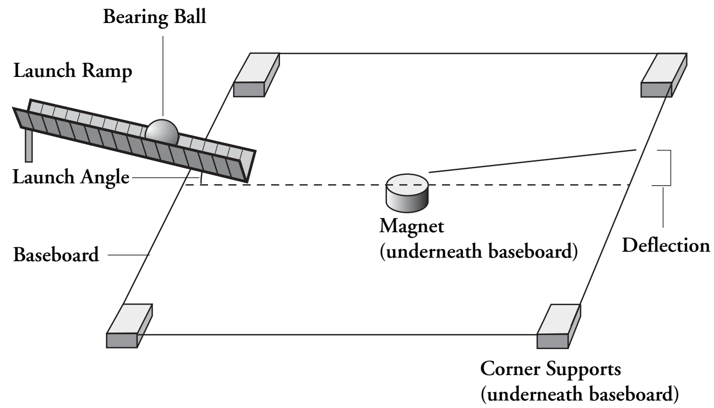


Figure 5. The experiment setup for magnetism-governed flybys of steel balls. An inclined launch ramp is used to guide a steel ball towards a magnet.

angle. Minimize the distance that the ball might drop (instead of rolling smoothly) from the ramp onto the baseboard.

Starting from the middle of the ramp, roll a bearing ball down the launcher. Experiment with the aiming of the launcher, watching the effect of close roll-bys of the balls. Close to the magnet the balls will be pulled off their straight-line paths. Far from the magnet, and going directly over the magnet's center, there will be no noticeable deviation of the path. (The lack of deviation directly over the magnet's center is the result of forces from all parts of the magnet acting equally on the ball. Similarly, there would be no deviation for an object falling through the center of a planet, if that were possible.)

The speed of the ball can be increased by steepening the ramp angle and/or by allowing more rolling distance on the ramp. The deviation will decrease for a faster "flyby" of the magnet. A larger ball with the same launch angle and position and range from the magnet will have less deviation than a small ball. The deviation is a function of the momentum (mass × velocity) of the ball, just as it is for a spacecraft.

Set up the launch ramp so that the magnet deviates a larger ball's path by a noticeable amount. Fix the launch ramp's position and angle and roll balls of varying size down the ramp from the same starting point. Note the amount of bending as a function of ball size or weight (mass). Ideally, a larger ball with the same launch angle and position and range from the magnet will have less deviation than a small ball. The momentum of a larger ball is greater than for a smaller ball by the ratio of the masses. Note that the difference in deviation for large and small spacecraft is too small to measure. Planets are so much more massive than any spacecraft that the difference in behavior of the spacecraft is undetectable.

Set up the launch ramp so that a ball's path deviates by some convenient, measurable amount. Fix the launch ramp's position and angle and roll balls down the ramp from different, marked starting points (see Figure 5). If the speed is changed by changing the length of the roll down the launcher, the ratio of the speeds is proportional to the ratio of the square root of the ratio of the roll lengths:

(new speed)/(old speed) = sqrt[(new length)/(old length)]

The launcher angle, with fixed launch point, can be varied for the same effect. The effect on the ball's rolling speed by changing the launcher angle is seen in the ratio of the sines of the old angle and the new angle:

(new speed)/(old speed) = [sin(new angle)]/[sin(old angle)]

This has assumed the magnet is a simple, short bar magnet, ideally with one pole projecting normal to the baseboard. The basic effect of deviating the motion of the balls is true for any common magnet (i.e., a dipole or a horseshoe) for paths staying outside the cross-section of the magnet. The motions get considerably more complex for balls, especially slow-rollers, that cross over a horseshoe magnet. In essence, two magnets are now affecting the motion, and that motion can demonstrate some extreme effects.

In Vol. 1, Chapter 9–7 of *The Feynman Lectures on Physics* (Addison-Wesley Publishing Co., Reading, MA, 1963), Feynman, Leighton, and Sands present a calculation of the motion of a planet around the Sun. Such a step-by-step calculation can be modified to illustrate a gravity-assist flyby of a planet.

2. Effect on Trajectory Demonstration

The path of a spacecraft before and after a planetary flyby closely follows straight lines at angles to each other (the asymptotes in Figure 3). The path can be mimicked with a beam from a laser pointer. Because spacecraft motion is along an asymptote before and after the flyby, it can be modeled with a reflection of the laser beam off a mirror (Figure 6). A flashlight could be used, but the spread in the beam makes the effect more difficult to observe and measure and a larger mirror is necessary.)

Put a large piece of wrapping paper on the experimental surface (table or floor). Place a mark on the paper to show the position of the laser and its beam direction. The position of the mirror should be fixed, and a line should be drawn on the paper at the mirror's position showing its orientation. Aim the laser from its position mark towards the mirror and mark on the paper the position of the beam's interception of the mirror. Using a piece of cardboard as a screen, note the position on the base paper of the reflected beam's "end" point on the screen.

CAUTION: BE CAREFUL NOT TO SHINE THE LASER BEAM INTO ANYONE'S EYES. EYE DAMAGE CAN RESULT.

Draw lines from the laser source point to the mirror intersection point, and from the mirror point to the end point indicated by the cardboard screen. Use a protractor to measure the angles of the inbound and outbound light with respect to a perpendicular from the mirror — they should be equal. Try several different angles to confirm that (angle in) = (angle out); i.e., the (angle of incidence) = (angle of reflection).

Rubber "super" balls can be horizontally bounced (reflected) off a hard vertical surface. Students can mark the launch

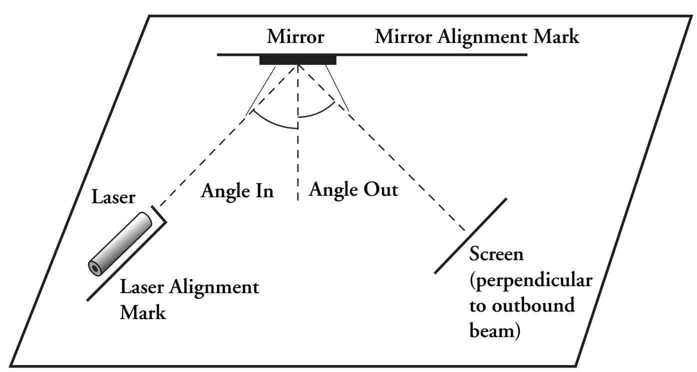


Figure 6. The experimental setup for the before/after effect of a planetary flyby. The laser beam reflects off the mirror to the screen. Measurement will show that (angle in) = (angle out) for all angles. To use rubber balls, the laser can be replaced with a v-shaped launch ramp and the mirror must be replaced with a solid, immovable wall.

point on the wrapping paper, the impact point on the surface, and the point where the ball is stopped. A launch ramp made of angle aluminum will allow better control and repeatability of the ball launches. Again, by connecting the points the angles can be measured.

To demonstrate the change in velocity relative to the Sun, place an X anywhere on the wrapping paper. Mark off some convenient length of line segment on the inbound leg and the outbound leg; both segments should be equal in length. Now draw a line from the symbolic Sun at X to the beginning of each line segment ("beginning" means the end closer to the starting point on the inbound leg). Resolve the two segments into vector components parallel to and perpendicular to the line (called a radius vector) from the Sun symbol (see Figure 4). Measurements of the lengths of the components parallel to the radius vector will be different for the inbound and outbound legs, establishing a change in velocity relative to the Sun, just as happens during a flyby maneuver. This shows why planetary flybys are used: the speed of the spacecraft relative to the Sun is increased, which decreases the travel time to the planet. In the case of gravity assist using Titan, the speed relative to Saturn is changed, with the goal of modifying Cassini's orbit around the planet.

Students can demonstrate the Pythagorean theorem by squaring the lengths of the radial and tangential components, summing them, and taking the square root of the sum. The result should equal the length of the original line segments.

Extensions

1. Make a small change in the angle and observe its effect on the end point. While the inbound and outbound angles are the same, the end point moves by a calculable amount. If the inbound angle is changed less than about 8 degrees, the shift in the position of the end point can be calculated with this formula:

shift in mm = [({1st angle in} - {2nd angle in})/57.3] × [mirror to screen distance in mm]

2. Use the same pair of incidence angles with the screen moved farther from the mirror. The ratio of the new and old shifts is directly proportional to the ratio of the new and old screen distances, an example of similar triangles (Figure 7). This demonstrates that precise pointing is necessary, as in billiards, to assure that a target will be hit.

With both laser+mirror and bouncing balls, the straight line paths before and after are like those of a spacecraft before and after its planetary flyby. The bending due to gravity is a sweeping curve and not the instantaneous reflection demonstrated here.

3. A better analogy to Cassini's interplanetary trajectory would use four small mirrors mounted on the sides of a box or the walls of a room. (**Reminder: Be very careful to avoid shining the laser into anyone's eyes.**) Precise aiming is necessary to reach the final end point. Even a small offset at the first mirror will make it difficult to reach the desired endpoint. To set up this demonstration, it is best to fix the beam's direction, place the first mirror, place the second mirror where the beam's first reflection intersects the wall, place the third mirror where the beam's second reflection intersects the wall, and place the fourth mirror where the beam's third reflection intersects the wall. A target can be placed where the beam's fourth reflection strikes the wall. Placing the mirrors at random and then trying to aim the laser so the beam strikes them all will be much more challenging.

Carefully offsetting the laser's aim at the first mirror will immediately affect the beam's position on the target, and will

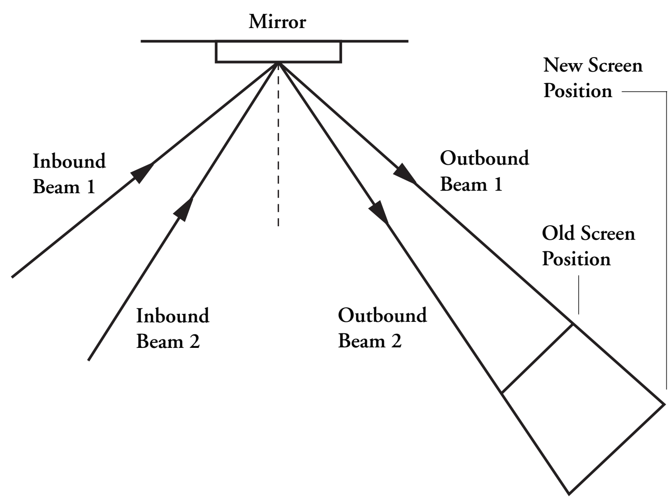


Figure 7. The separation of laser beam endpoints grows if the screen is moved farther from the mirror. By analogy, a small change in a planetary flyby can produce large effects later.

likely remove it completely since the beam will probably be missing one or more of mirrors 2, 3, or 4. Note that the room will have to be dark for both the setup and demonstration. Figure 8 shows an experiment setup.

A billiards table is a small and easily controlled environment, though any player will tell you that getting the desired ball into a desired pocket takes careful aiming and hitting. Interplanetary space flight in the solar system takes careful aiming and precise launch power. But different from billiards, we can correct the spacecraft's motion along the way. The gravitational field of the Sun and planets are all well known and their effects are easily calculated. Small imperfections in direction and speed during the launch of the rocket can be measured.

Just as a car must be steered to stay in a lane on the road, so can the motion of a spacecraft be corrected along its trajectory, but the corrections necessary are much smaller and are needed much less frequently than maintaining a car in a lane. These corrections are made to make sure that desired aim points — necessary for a successful planetary flyby — are passed. Trajectory correction maneuvers are scheduled throughout the course of an interplanetary trip and during Cassini's Saturn tour as well to make sure the spacecraft stays precisely in the middle of the "lane."

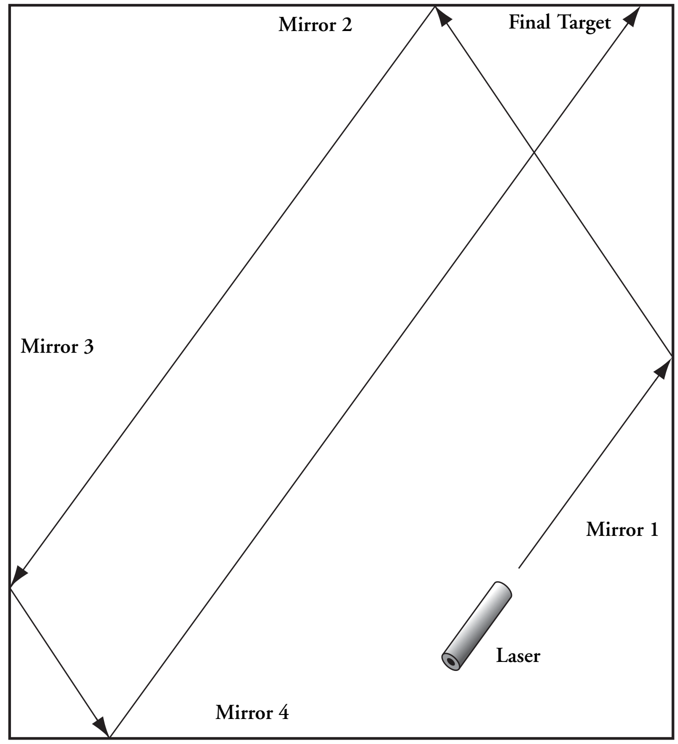


Figure 8. Mirrors placed on walls (or the sides of a box) throw a laser beam around the room to a final target. A small offset of the laser's pointing will affect the final position of the beam at the end of the string of reflections. Each individual reflection is like a planetary flyby. Any set of four reflections (around the room as illustrated, zigzag, etc.) will demonstrate the effect of changing the laser's pointing. Be very careful to avoid shining the laser into anyone's eyes. The laser can be set up so its beam is high above students' eye levels, or the whole demonstration can be performed inside a large cardboard box.

Education Standards

A visit to the URL http://www.mcrel.org yielded the following national education standards and included benchmarks that may be applicable to this activity.

Mathematics Standards

1. Uses a variety of strategies in the problem-solving process.

LEVEL 1 (GRADES K–2)

Draws pictures to represent problems.

Uses discussions with teachers and other students to understand problems.

Makes organized lists or tables of information necessary for solving a problem.

4. Understands and applies basic and advanced properties of the concepts of measurement.

LEVEL 2 (GRADES 3–5)

Understands the basic measures perimeter, area, volume, capacity, mass, angle, and circumference.

Selects and uses appropriate tools for given measurement situations (e.g., rulers for length, measuring cups for capacity, protractors for angle).

Understands that measurement is not exact (i.e., measurements may give slightly different numbers when measured multiple times).

LEVEL 3 (GRADES 6–8)

Understands the concepts of precision and significant digits as they relate to measurement (e.g., how units indicate precision).

Selects and uses appropriate units and tools, depending on degree of accuracy required, to find measurements units indicate precision for real-world problems.

5. Understands and applies basic and advanced properties of the concepts of geometry.

LEVEL 1 (GRADES K–2)

Understands that geometric shapes are useful for representing and describing real world situations.

LEVEL 2 (GRADES 3–5)

Understands characteristics of lines (e.g., parallel, perpendicular, intersecting) and angles (e.g., right, acute).

9. Understands the general nature and uses of mathematics.

LEVEL 2 (GRADES 3–5)

Understands that numbers and the operations performed on them can be used to describe things in the real world and predict what might occur.

Understands that mathematical ideas and concepts can be represented concretely, graphically, and symbolically.

LEVEL 4 (GRADES 9–12)

Understands that mathematics provides a precise system to describe objects, events, and relationships and to construct logical arguments.

Science Standards

3. Understands the composition and structure of the universe and Earth's place in it.

LEVEL 3 (GRADES 6–8)

Knows that gravitational force keeps planets in orbit around the Sun and moons in orbit around the planets.

9. Understands the sources and properties of energy.

LEVEL 2 (GRADES 3–5)

Knows that light can be reflected, refracted, or absorbed.

LEVEL 3 (GRADES 6–8)

Knows ways in which light interacts with matter (e.g., transmission, including refraction; absorption; scattering, including reflection).

10. Understands forces and motion.

LEVEL 1 (GRADES K–2)

Knows that magnets can be used to make some things move without being touched.

Knows that things near Earth fall to the ground unless something holds them up.

Knows that the position of an object can be described by locating it relative to another object or the background.

Knows that the position and motion of an object can be changed by pushing or pulling.

Knows that things move in many different ways (e.g., straight line, zigzag, vibration, circular motion).

LEVEL 2 (GRADES 3–5)

Knows that magnets attract and repel each other and attract certain kinds of other materials (e.g., iron, steel).

Knows that Earth's gravity pulls any object toward it without touching it.

Knows that an object's motion can be described by tracing and measuring its position over time.

Knows that when a force is applied to an object, the object either speeds up, slows down, or goes in a different direction.

Knows the relationship between the strength of a force and its effect on an object (e.g., the greater the force, the greater the change in motion; the more massive the object, the smaller the effect of a given force).

LEVEL 3 (GRADES 6–8)

Understands general concepts related to gravitational force (e.g., every object exerts gravitational force on every other object; this force depends on the mass of the objects and their distance from one another; gravitational force is hard to detect unless at least one of the objects, such as Earth, has a lot of mass).

Knows that an object's motion can be described and represented graphically according to its position, direction of motion, and speed.

Understands effects of balanced and unbalanced forces on an object's motion (e.g., if more than one force acts on an object along a straight line, then the forces will reinforce or cancel one another, depending on their direction and magnitude; unbalanced forces such as friction will cause changes in the speed or direction on an object's motion).

Knows that an object that is not being subjected to a force will continue to move at a constant speed and in a straight line.

LEVEL 4 (GRADES 9–12)

Knows that the strength of the gravitational force between two masses is proportional to the masses and inversely proportional to the square of the distance between them.

Knows that laws of motion can be used to determine the effects of forces on the motion of objects (e.g., objects change their motion only when a net force is applied; whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted on the first object; the magnitude of the change in motion can be calculated using the relationship $F = ma$, which is independent of the nature of the force).

12. Understands the nature of scientific inquiry.

LEVEL 1 (GRADES K–2)

Knows that learning can come from careful observations and simple experiments.

Knows that tools (e.g., thermometers, magnifiers, rulers, balances) can be used to gather information and extend the senses.

LEVEL 2 (GRADES 3–5)

Knows that scientists use different kinds of investigations (e.g., naturalistic observation of things or events, data collection, controlled experiments), depending on the questions they are trying to answer.

Plans and conducts simple investigations (e.g., formulates a testable question, makes systematic observations, develops logical conclusions).

Uses appropriate tools and simple equipment (e.g., thermometers, magnifiers, microscopes, calculators, graduated cylinders) to gather scientific data and extend the senses.

LEVEL 3 (GRADES 6–8)

Designs and conducts a scientific investigation (e.g., formulates hypotheses, designs and executes investigations, interprets data, synthesizes evidence into explanations, proposes alternative explanations for observations, critiques explanations and procedures).

Uses appropriate tools (including computer hardware and software) and techniques to gather, analyze, and interpret scientific data.

Establishes relationships based on evidence and logical argument (e.g., provides causes for effects).

LEVEL 4 (GRADES 9–12)

Designs and conducts scientific investigations (e.g., formulates testable hypotheses; identifies and clarifies the method, controls, and variables; organizes, displays, and analyzes data; revises methods and explanations; presents results; receives critical response from others).

Uses technology (e.g., hand tools, measuring instruments, calculators, computers) and mathematics (e.g., measurement, formulas, charts, graphs) to perform accurate scientific investigations and communications.

***Teachers** — Please take a moment to evaluate this product at*
http://ehb2.gsfc.nasa.gov/edcats/educational_brief.
Your evaluation and suggestions are vital to continually improving NASA educational materials. Thank you.

Student Worksheet — Planetary Billiards

Procedure

Trajectory Modification Demonstration

Observe the motion of the bearing balls as they roll over the magnet.

- Is the deflection greater for larger or smaller bearing balls?
- What parameter of the rolling ball changes when the ball is released from the top of the ramp or from the middle of the ramp? How does this affect the deflection caused by the magnet?
- What parameter of the rolling ball changes when the ramp is made steeper? How does this affect the deflection caused by the magnet?
- If a bearing ball rolls directly over the center of the magnet, is it deflected? Why or why not?
- If a bearing ball is rolled very slowly across the magnet it may gyrate around and finally come to rest directly over the magnet instead of continuing on as faster-moving bearing balls or a spacecraft going by a planet would. Why is the behavior different?

Effect on Trajectory Demonstration

Put a large piece of wrapping paper on the experimental surface (table or floor). Place a mark on the paper to show the position of the laser and its beam direction. The position of the mirror should be fixed, and a line should be drawn on the paper at the mirror's position showing its orientation. Aim the laser from its position mark towards the mirror and mark on the paper the position of the beam's interception of the mirror. Using a piece of cardboard as a screen, note the position on the base paper of the reflected beam's "end" point on the screen.

CAUTION: BE CAREFUL NOT TO SHINE THE LASER BEAM INTO ANYONE'S EYES. EYE DAMAGE CAN RESULT.

Draw lines from the source point to the mirror intersection point, and from the mirror point to the end point. Use a protractor to measure the angles of the inbound and outbound light with respect to a perpendicular from the mirror. Change the position of the laser and repeat the measurements. Try this for several different angles.

- Do your measurements confirm that (angle in) = (angle out) — i.e., the (angle of incidence) = (angle of reflection)?
- Demonstrate the change in velocity relative to the Sun for one (or more) reflection experiments. Place an X anywhere on the wrapping paper. Mark off some convenient length of line segment on the inbound leg and the outbound leg; the segments should be equal in length. Now draw a line from the symbolic Sun at X to the beginning of each line segment (beginning means the end closer to the starting point on the inbound leg). Resolve the two segments into vector components parallel to and perpendicular to the line (called a radius vector) from the Sun symbol. Measurements of the lengths of the components parallel to the radius vector will be different for the inbound and outbound legs, establishing a change in velocity relative to the Sun, just as happens during a flyby maneuver. This shows why planetary flybys are used: the speed of the spacecraft relative to the Sun is increased, which decreases the travel time to the planet. In the case of gravity assist using Titan, the speed relative to Saturn is changed, with the goal of modifying Cassini's orbit around the planet.
- Measure the lengths of the vector components and the line segments for the inbound and outbound legs. Test whether the values match the Pythagorean theorem ($x^2 + y^2 = z^2$). If the results don't match, what could be causing the error(s)?